Student Name: ______________________

Student Number: | | | | | | | | | |

Teacher Name: ______________________

TEMPUS CELERIUS RADIO FUGIT

ABBOTSLEIGH

2021

HIGHER SCHOOL CERTIFICATE

Assessment Task 4

Mathematics Advanced

General Instructions

- Reading time – 10 minutes
- Working time – 3 hours
- Write using black pen only.
- **NESA approved** calculators may be used.
- **NESA approved** reference sheet is provided.
- Make sure your HSC candidate number is on the front of each question.
- Answer Section I Multiple-Choice questions on the answer sheet provided.
- Answer Section II questions in the space provided.
- There is a spare page for extra working at the end of each question in Section II.
- In Questions 11 - 16, show relevant mathematical reasoning and/ or calculations
- Marks may not be awarded for careless or badly arranged working.
- Marks are an indicator of required working.

Total Marks – 100

- Attempt Sections I and II

Section I Pages 2 - 6

10 marks

- Attempt Questions 1–10
- Allow about 15 minutes for this section.

Section II Pages 7 - 16

90 marks

- Attempt Questions 11 - 16
- Allow about 2 hour 45 minutes for this section.

Section I (10 marks)

Attempt Questions 1 – 10
Use the multiple-choice answer sheet
Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample $2 + 4 =$ (A) 2 (B) 6 (C) 8 (D) 9

(A) ○ (B) ● (C) ○ (D) ○

If you think you've made a mistake, put a cross through the incorrect answer and fill in the new answer.

(A) ● (B) ~~●~~ (C) ○ (D) ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate this by writing the word *correct* and drawing an arrow as follows.

correct

(A) ~~●~~ (B) ~~●~~ (C) ○ (D) ○

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1. What is the value of x for which $9^{3-5x} = 27^{2x}$?

A. $\frac{1}{3}$

B. $-\frac{3}{7}$

C. $\frac{3}{8}$

D. $-\frac{3}{8}$

2. What is the value $\tan x$ of given that $\sin x = \frac{2}{5}$ for $\frac{\pi}{2} < x < \pi$.

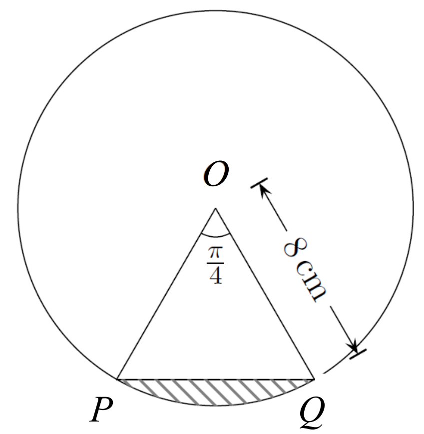
A. $-\frac{\sqrt{21}}{2}$

B. $-\frac{5}{\sqrt{21}}$

C. $\frac{\sqrt{21}}{5}$

D. $-\frac{2}{\sqrt{21}}$

3. Given that O is the centre of the circle and $OQ = 8cm$, what is the area of the shaded segment?



NOT TO
SCALE

A. $64\left(\frac{\pi}{4}-\frac{1}{\sqrt{2}}\right)$ cm^2

B. $32\left(\frac{\pi}{4}-\frac{1}{\sqrt{2}}\right)$ cm^2

C. $16\left(\frac{\pi}{4}+\frac{1}{\sqrt{2}}\right)$ cm^2

D. $32\left(\frac{1}{\sqrt{2}}-\frac{\pi}{4}\right)$ cm^2

4. Find the exact value of $\int_0^8 \sqrt{64-x^2}\ dx$

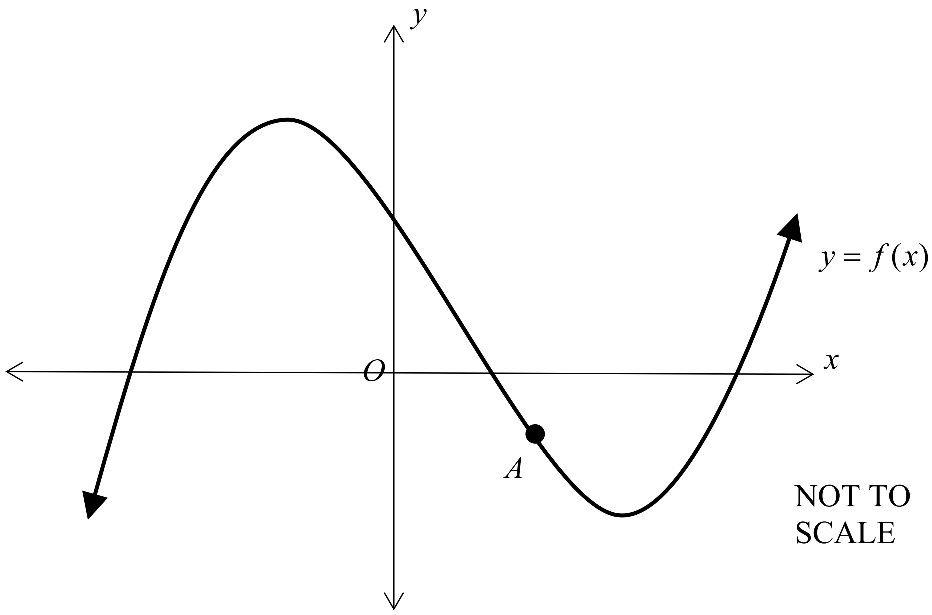
A. 32 units^2

B. 32π units^2

C. 16 units^2

D. 16π units^2

5. The graph of the relation $y = f(x)$ is shown below.



What statement is correct at point A?

A. $f'(x) > 0$ and $f''(x) < 0$

B. $f'(x) < 0$ and $f''(x) > 0$

C. $f'(x) > 0$ and $f''(x) > 0$

D. $f'(x) < 0$ and $f''(x) < 0$

6. What is the domain and range of a circle with the equation $x^2 + 4x + y^2 = 5$?

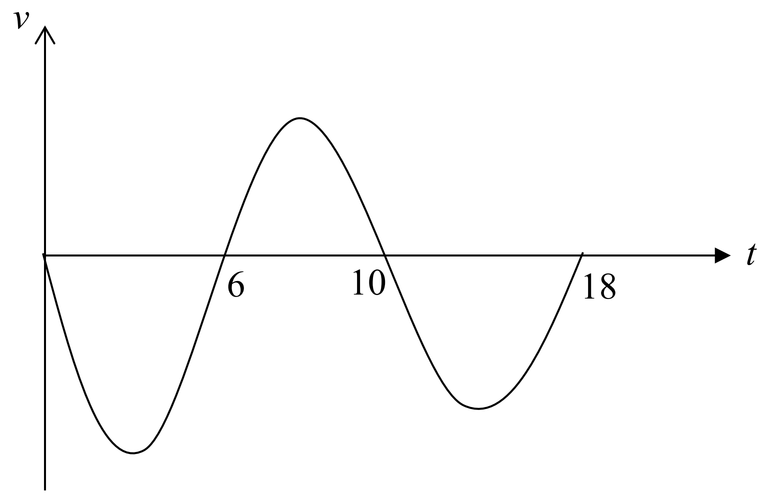
A. domain $\left[-2-\sqrt{5}, -2+\sqrt{5}\right]$; range $\left[-\sqrt{5}, \sqrt{5}\right]$

B. domain $\left[2-\sqrt{5}, 2+\sqrt{5}\right]$; range $\left[-\sqrt{5}, \sqrt{5}\right]$

C. domain $[-1, 5]$; range $[-3, 3]$

D. domain $[-5, 1]$; range $[-3, 3]$

7. A particle moves along a straight line. Its velocity v at time t is shown in the graph below.



For what value of t is the displacement of the particle a maximum?

A. 18

B. 10

C. 8

D. 6

8. What is the change in amplitude and period when the function $f(x) = \frac{1}{2}\sin 6x$ is transformed into $g(x) = \sin 3x$.

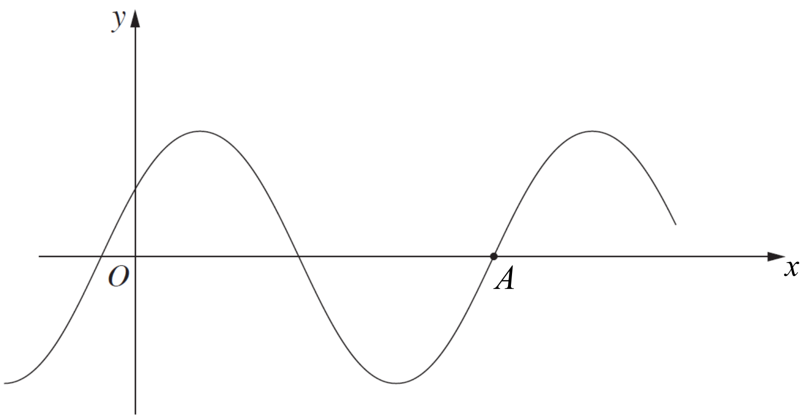
A. The amplitude is halved and the period is halved.

B. The amplitude is halved and the period is doubled.

C. The amplitude is doubled and the period is doubled.

D. The amplitude is doubled and the period is halved.

9. The graph of the function $y=\cos\left(2\left(x-\frac{\pi}{6}\right)\right)$ is shown below.



What are the coordinates of the point A ?

A. $\left(\frac{5\pi}{12},0\right)$

B. $\left(\frac{2\pi}{3},0\right)$

C. $\left(\frac{11\pi}{12},0\right)$

D. $\left(\frac{7\pi}{6},0\right)$

10. Which of the following statements is true for the function $f(x)=e^{|x|}-1$.

A. The function is an even function.

B. The function is not continuous at $x=0$.

C. The function has a stationary point at $x=0$.

D. The function has an asymptote at $y=-1$.

End of Section I

2021 HSC Course

Assessment Task 4

Student Number:

Mathematics Advanced

Section II - 90 marks

Attempt Questions 11-16

Allow about 2 hour and 45 minutes for this section

Answer the questions in the spaces provided.
These spaces provide guidance for the expected length of response.
Your responses should include relevant mathematical reasoning and/or calculations.

Extra writing space is provided at the end of each question. If you use this space, clearly indicate which question you are answering

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Question 11 (15 marks) **Marks**

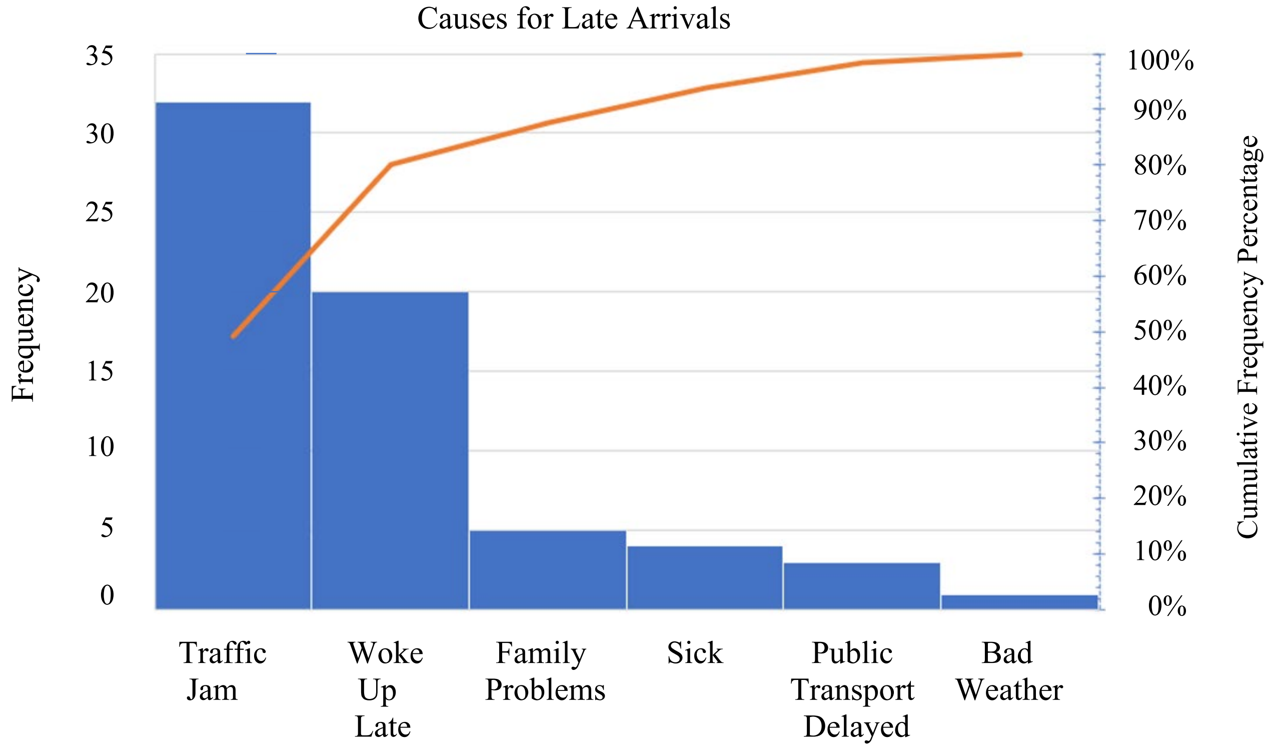
(a) What is the value of $e^3 \tan(1.5)$ correct to 4 significant figures. **1**

(b) Simplify $1+\tan^2 x$. **1**

(c) Find the exact value of $\int_0^4 \frac{2x}{x^2+8}\,dx$. **2**

(d) Consider the Pareto chart below.

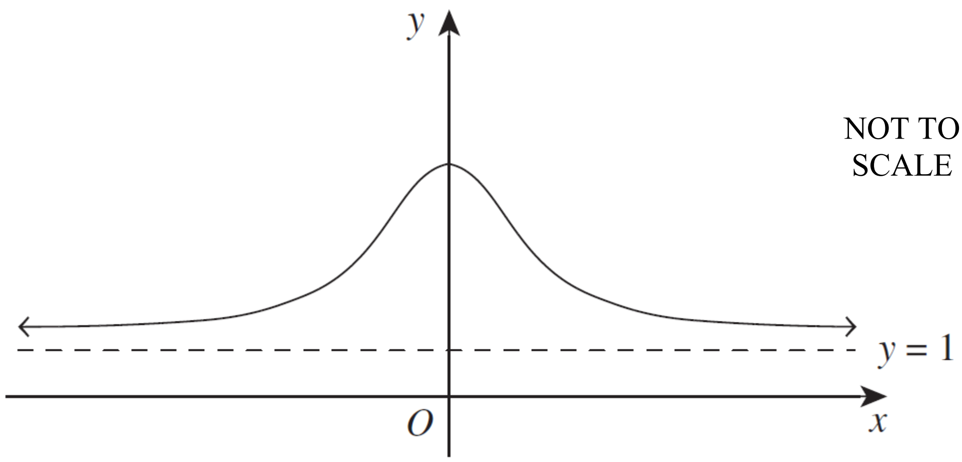
What percentage of late arrivals were due to waking up late? **1**



Question 11 continues on next page

Question 11 (continued)

(e) The graph shows the function $y = f(x)$ with a horizontal asymptote at $y = 1$. **2**

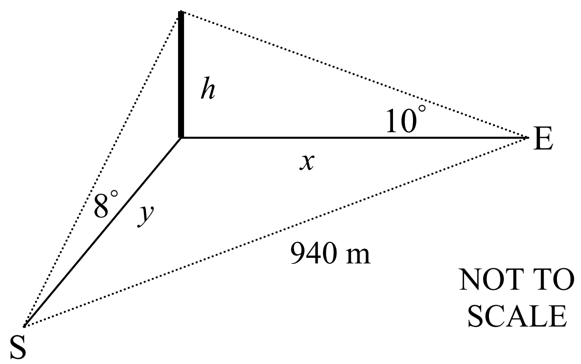


In the space provided on your answer sketch he graph of $y = f'(x)$.

(f) Find $\dfrac{d}{dx}\left(\ln\left(\dfrac{\sin x}{1+\cos x}\right)\right)$. Give your answer in simplest form. **4**

(Hint: Use your log laws to help with the differentiation).

(g) A surveyor who is y metres due south of a tower sees the top of it with an angle of elevation 8°. A second surveyor is x metres due east of the tower. From her position the angle of elevation is 10° to the top of the tower. The two surveyors are 940 m apart.



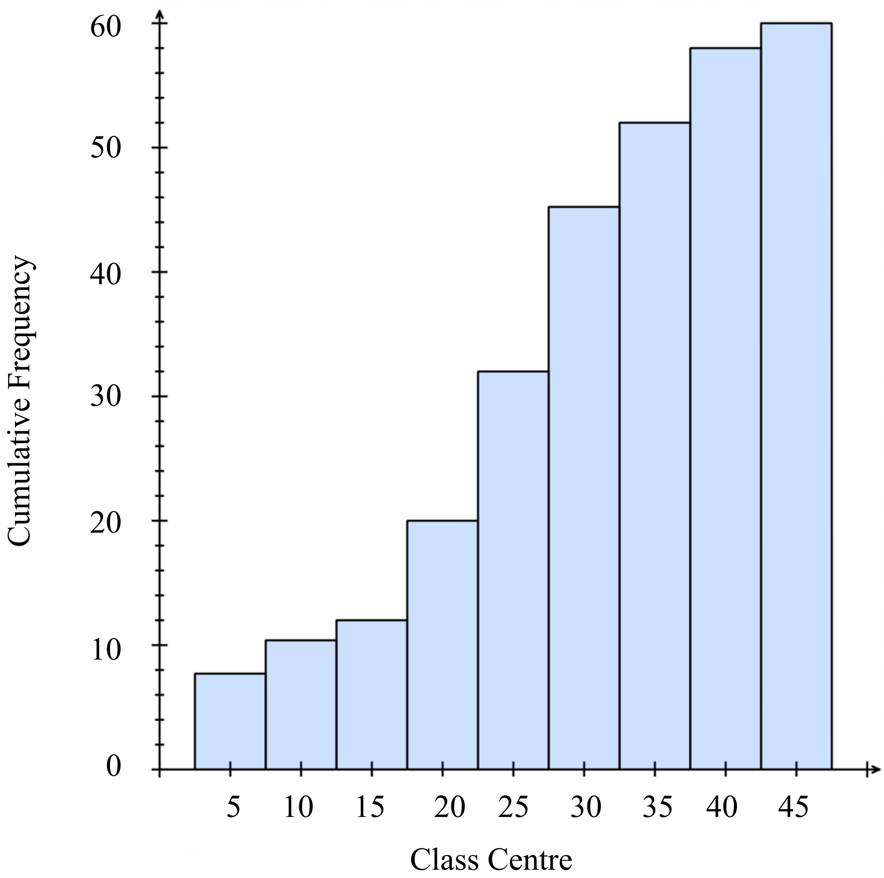
(i) Show that $y = \dfrac{h}{\tan 8^\circ}$. **1**

(ii) Find the height (h) of the tower to the nearest metre. **3**

End of Question 11

Question 12 (15 marks) **Student Number:**

(a) A cumulative frequency histogram is shown below.



(i) On the graph provided in your answer sheet, construct a cumulative frequency histogram (ogive). **1**

(ii) Estimate the median of the data. **1**

(b) If $\int_0^3 f(x)\,dx = 8$, find $\int_0^3 f(x+4)\,dx$. **2**

Question 12 continues on next page

Question 12 (continued)

(c) A survey of 30 people found that 21 like AFL and 12 like soccer.
Also 7 people like both AFL and soccer and 4 like neither AFL nor soccer.

(i) Using the information, complete the Venn diagram on your answer sheet. **1**

(ii) If one of the 30 people was randomly selected, find the probability. that they like soccer only. Give your answer in simplest form. **1**

(d) From a group of 15 hockey players at a game of hockey, 13 played on the field, 7 sat on the bench and 5 both played and sat on the bench. A hockey player is chosen at random from the team. **2**
Let A be the event 'the person played on the field' and let B be the event 'the person sat on the bench'.

Find $P(A \mid B)$.

(e) Twenty students sit a Geography test and the mean of their scores is 78. Two students sit the test late and their scores are 96 and 82. **1**

What is the new mean for the Geography test?

(f) Describe the three transformation which, when applied in succession, change the graph of $y = \sqrt{x}$ to the graph with the equation $y = 4\sqrt{x-1} + 3$. **3**

(g) A function is defined by $f(x) = \dfrac{2x-1}{3x+2}$. **3**

Find the gradient of the normal at the point where $x = -1$.

End of Question 12

Question 13 (15 marks) **Student Number:** | | | | | | | | |

(a) An adaptive learning tool has been designed to assist students with their studies in a course. The amount of time (t) some of the students spent using the tool and the number of tasks they completed (n) are shown in the table.

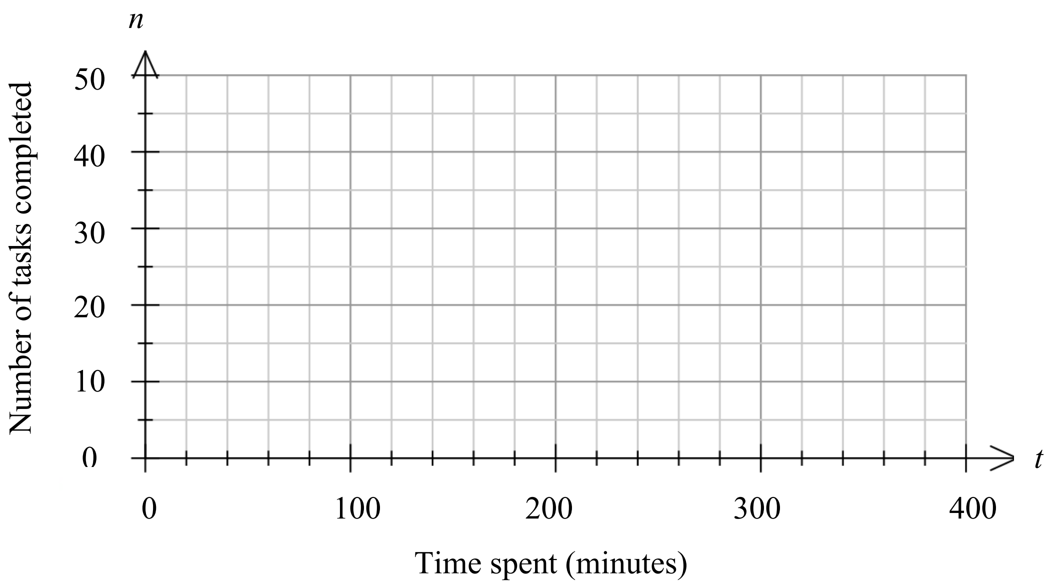
Student Name	Time spent (minutes)	Number of tasks completed
Fred Flintstone	5	3
Wilma Flintstone	94	10
Barney Rubble	66	28
Betty Rubble	17	23
Pebbles Flintstone	340	32
Bamm-Bamm Rubble	223	40
Dino	38	16
Mr Slate	171	35

(i) Calculate Pearson's coefficient for the data, correct to 4 decimal places. **1**

(ii) Identify the direction and the strength of the linear association between time spent and number of tasks completed. **1**

(iii) Write the equation of the least-squared regression line in terms of the variables correct to 3 decimal places. **2**

(iv) On the grid provided in your booklet, sketch the least-squared regression line for this data. **1**



Question 13 continues on next page

(v) If a student completes 27 tasks, what is the time spent using the adaptive tool? Give your answer to the nearest minute. **1**

(vi) Is finding the time using the adaptive tool for 27 tasks an example of extrapolation or interpolation? Explain **1**

(b) The discrete random variable X has the following probability distribution.

X	0	1	2
$P(X=x)$	a	b	0.3

Given that $E(X)=0.8$, then

(i) Find the values of a and b. **2**

(ii) Calculate the Var(X) correct to 2 decimal places. Show all working. **2**

(c) Whilst performing some experiments, researchers have found that the rate of change of the distance (in cm/s) travelled by a particle is given by $A(t)=40\ln(1+5t)$, where t is the number of seconds elapsed since the start of the research.

Let D cm be the distance travelled by the particle from $t=0.1$ to $t=0.5$.

(i) Estimate D by using the trapezoidal rule with 4 sub-intervals. **2**

(ii) Is D an over-estimate or an under-estimate? Justify your answer. **2**

End of Question 13

Question 14 (15 marks) **Student Number:**

(a) A function is given by $f(x) = 4x^2 - x^4$.

(i) By solving $f(x) = 0$ show that x-intercepts are -2, 0 and 2. **2**

(ii) Prove that $f(x)$ is an even function. **2**

(iii) Show that $f'\left(\sqrt{2}\right) = 0$. **1**

(iv) Given that $\left(\sqrt{2}, 4\right)$ is a maximum turning point, explain in words why $\left(-\sqrt{2}, 4\right)$ is also a maximum turning point. **1**

(b) A city is suffering through a drought and adopts a plan to import water from another city. It is given that the volume of water imported in the 1st year since the start of the plan is 1.8×10^9 m^3. In subsequent years, the volume of water imported each year is 90% of the volume of water imported in the previous year

(i) Find how much water was imported in the 4th year. Give your answer correct 1 decimal place. **1**

(ii) Find the total volume of water imported in the first 10 years. Give your answer correct 1 decimal place. **3**

(iii) Explain why the total volume of water imported since the start of the plan will never exceed 1.9×10^{10} m^3. **2**

(c) Solve $e^{2\ln x} = 4x + 21$ **2**

(d) Evaluate $\displaystyle\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} 3\sin x \, dx$. **1**

End of Question 14

Question 15 (15 marks) **Student Number:**

(a) The displacement of an object measured in metres after t seconds is given by $x = e^{t^2} - 2t^2$.

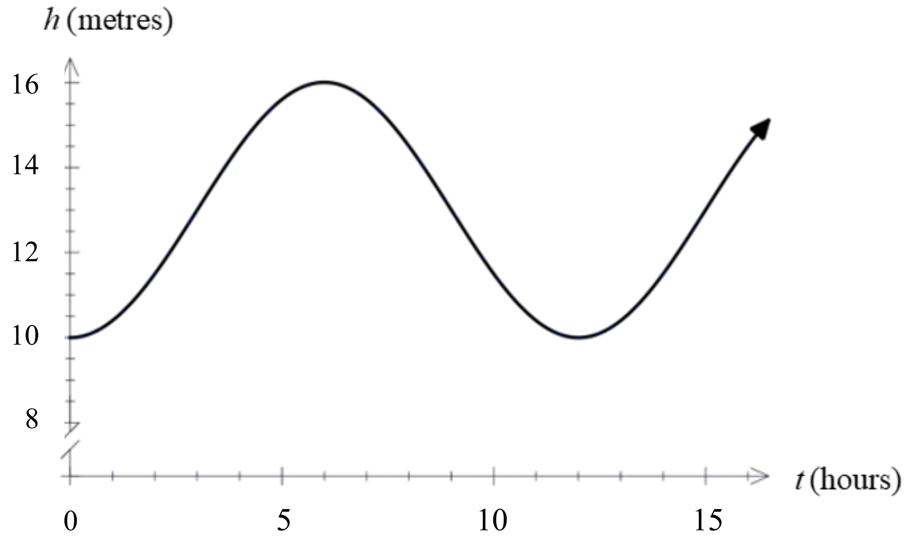
(i) Given that the velocity is the rate of change of the displacement, find an expression for the velocity. **2**

(ii) Find the two times when the object is stationary. **3**

(iii) Find an expression for the acceleration of the object. **2**

(b) The rise and fall in sea level in Batesman Bay due to tides, can be modelled by the cosine function below:

$$h(t) = A\cos(bt) + 13$$



At 8 am it is low tide, and the channel is 10 m deep. At 2 pm it is high tide, and the channel is 16 m deep. A ship needs 11.5 m of water to sail.

(i) Find the values of A and b. **2**

(ii) Between what times in the first day can the ship sail? **2**

Question 15 continues on next page

Question 15 (continued)

(c) The population of an ant colony is modelled by the equation $P = 1000e^{kt}$, where t is measured in weeks.

(i) Find the initial population. **1**

(ii) After 10 weeks the population is 25 000. **1**

Show that $k = 0.3219$, correct to 4 decimal places.

(iii) During which week will the population exceed 80 000 for the first time? **2**

End of Question 15

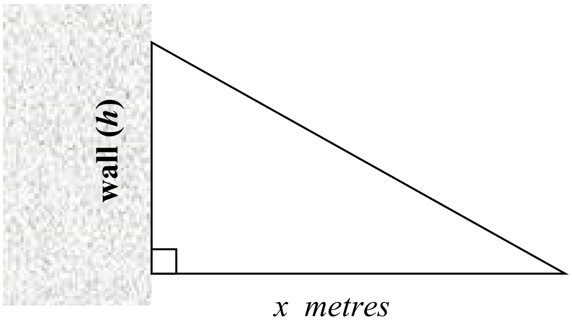
Question 16 (15 marks) **Student Number:**

(a) An artist posted a song online. Each day there were $2^n + 3n$ downloads, where n is the number of days after the song was posted.

(i) Find the number of downloads on the third day after the song was posted. **1**

(ii) What is the total number of times the song was downloaded in the first 20 days after it was posted? **3**

(b) An enclosure in the shape of a right-angled triangle is to be made using 18 m of fencing, with an existing wall serving as one side of the triangle, where no fencing is required. The area of the enclosure is A square metres where the side of the triangle perpendicular to the wall is x m in length.



(i) Show that the area is given by $A = 3x\sqrt{9-x}$. **2**

(ii) Find the value of x that will maximise the area, showing why this is the maximum. **4**

(c) A block of ice in the form of a cube melts in such a way that the block remains cubic. After a time t hours the volume of the block is V m^3. Initially the cube has edge length of 4 metres and after 9 hours the cube has edge length of 1 metre.

(i) If the **rate of change** of volume, R m^3 per hour, is given by $R = -k$ for some constant $k > 0$, show that $V = 64 - 7t$. **3**

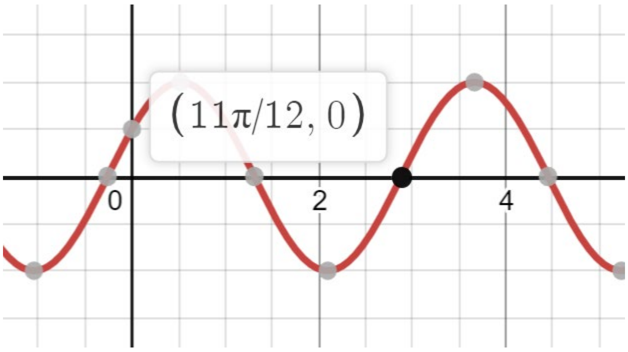
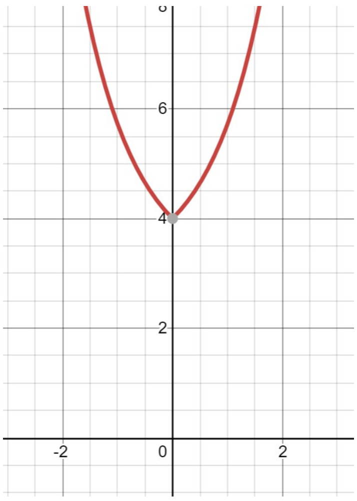
(ii) Find when the cube has edge length 2 metres. **2**

End of paper

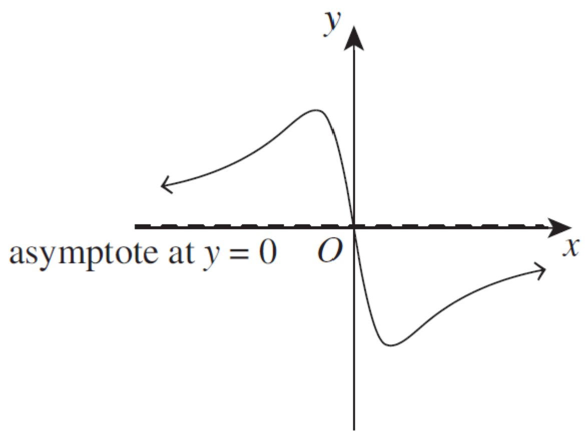
2021 Mathematics Advanced Trial Exam Solutions

Section 1

1.	$9^{3-5x} = 27^{2x}$ $3^{2(3-5x)} = 3^{3(2x)}$ $2(3-5x) = 3(2x)$ $6-10x = 6x$ $16x = 6$ $x = \frac{3}{8}$ $\quad\therefore C$
2.	$\sin x = \frac{2}{5}$ $x^2 + 2^2 = 5^2$ $x = \sqrt{21}$ $\therefore \tan x = -\frac{2}{\sqrt{21}} \quad for \quad \frac{\pi}{2} < x < \pi \quad \therefore D$
3.	$A = \frac{1}{2}r^2\theta - \frac{1}{2}ab\sin\theta$ $A = \frac{1}{2}\times 8^2 \times \frac{\pi}{4} - \frac{1}{2}\times 8\times 8\times \sin\frac{\pi}{4}$ $A = 32\left(\frac{\pi}{4} - \frac{1}{\sqrt{2}}\right) \quad \therefore B$
4.	$\int_0^8 \sqrt{64-x^2}\ dx = \frac{\pi\times 8^2}{4}$ $= 16\pi \quad \therefore D$
5.	Negative gradient and concave up $f'(x) < 0$ and $f''(x) > 0 \quad \therefore B$
6.	$x^2 + 10x + y^2 = 11$ $x^2 + 10x + 25 + y^2 = 11 + 25$ $(x+5)^2 + y^2 = 36$ $\therefore$ *Circle centre* $(-5,0)$ *radius* $= 6$ $\therefore$ *Domain* $-11 \le x \le 1$ *and Range* $-6 \le y \le 6 \quad \therefore C$

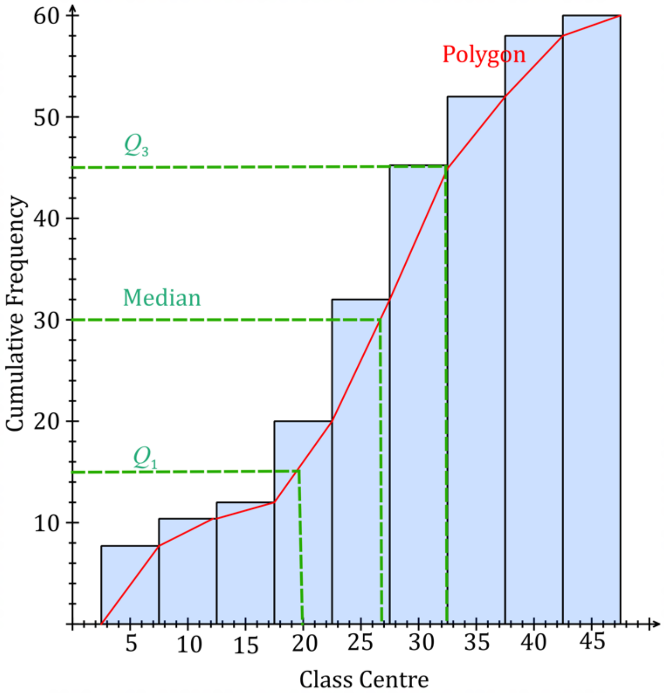
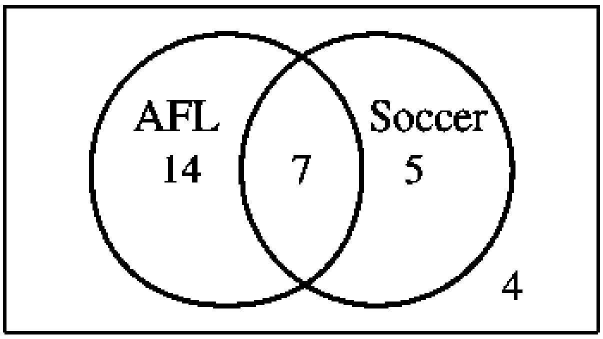
7.	Max when moves from v > 0 to v < 0 Therefore at $x = 10$ $\therefore B$
8.	$f(x) = \frac{1}{2}\sin 6x \;\Rightarrow\; g(x) = \sin 3x.$ $Amplitude\ \frac{1}{2} \Rightarrow 1 \quad \therefore doubled$ $Period\ \frac{2\pi}{6} = \frac{\pi}{3} \Rightarrow \frac{2\pi}{3} \quad \therefore\ doubled \qquad \therefore D$
9.	 $y = \cos\left(2\left(x - \frac{\pi}{6}\right)\right)$ has a period of $\frac{2\pi}{2} = \pi$ and translated $\frac{\pi}{6}$ to the right. Therefore crosses at $\frac{3\pi}{4} + \frac{\pi}{6} = \frac{11\pi}{12}$ $\therefore C$
10.	$f(x) = e^{\lvert x \rvert} + 3$  Is an even function $\therefore A$

Section II
Question 11

11.	(a)		$e^3 \tan(1.5) = 283.234591$ $= 283.2$ (correct to 4 sig figs) ☑ Important: Must be in radian mode
	(b)		$1 + \tan^2 x = sec^2 x$ ☑
	(c)		$\int_0^4 \frac{2x}{x^2+8} dx = \left[\ln(x^2+8) \right]_0^4$ ☑ $= \ln 24 - \ln 8$ $= \ln 3$ ☑
	(d)		80% - 49% = 31% ☑ (±3)
	(e)		The graph of $y = f(x)$ is: • increasing in the domain $(-\infty, 0)$ • decreasing in the domain $(0, \infty)$ • stationary at $x = 0$ • as $x \to \pm\infty$, $y \to 1$. Hence, the derivative $y = f'(x)$ must have the following properties. • above the x-axis in the domain $(-\infty, 0)$ • below the x-axis in the domain $(0, \infty)$ • x-intercept at $x = 0$ • as $x \to \pm\infty$, $f'(x) \to 0$  ☑ Shape and asymptote. Arrows mus direct towrads the x axis ☑ Stationary point becoming the x intercept at (0,0)

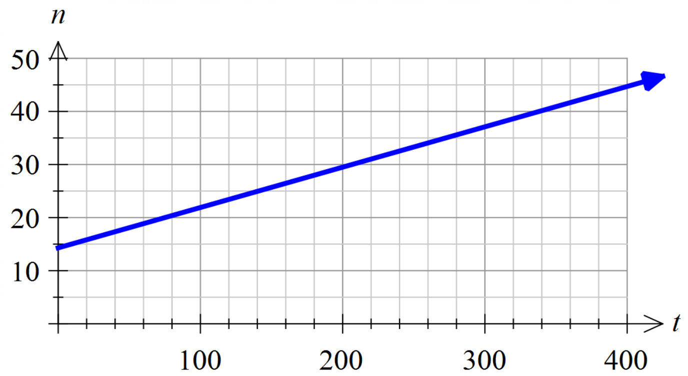
	(f)		$=\frac{d}{dx}\left(\ln(\sin x)-\ln(1+\cos x)\right)$ ✓ $=\frac{\cos x}{\sin x}-\frac{-\sin x}{1+\cos x}$ ✓ $=\frac{\cos x(1+\cos x)-\sin x(-\sin x)}{\sin x(1+\cos x)}$ $=\frac{\cos x+\cos^2 x+\sin^2 x}{\sin x(1+\cos x)}$ ✓ $=\frac{1+\cos x}{\sin x(1+\cos x}$ $=\frac{1}{\sin x}$ ✓ $=\text{cosec}x$
	(g)	(i)	$\tan 8^\circ=\frac{h}{y}$ $y=\frac{h}{\tan 8^\circ}$ ✓
	(h)	(ii)	$x=\frac{h}{\tan 10}$ $x^2+y^2=940^2$ (Δ is right angled) $\left(\frac{h}{\tan 8}\right)^2+\left(\frac{h}{\tan 10}\right)^2=940^2$ ✓ $h^2(\cot^2 8^\circ+\cot^2 10^\circ)=940^2$ $h=\sqrt{\frac{940^2}{\cot^2 8^\circ+\cot^2 10^\circ}}$ ✓ $=103.3\text{ m}$ $= 103$ *metres* (*to the nearest metre*) ✓

Question 12

12.	(a)	(i)	 ☑ for the ogive (red line)
		(ii)	Median = 27 ☑ (± 1), 25 is not acceptable, as it is continuous data
	(b)		$\int_0^3 f(x)\ dx = 8$ If $\therefore \int_{-4}^{-1} f(x+4)\ dx = 8$ ☑ Needed more information about the function to solve between $x = 0$ and $x = 3$ for $f(x+4)$ ☑
	(c)	(i)	 ☑
		(ii)	$P(soccer\ only) = \frac{5}{30}$ $= \frac{1}{6}$ ☑

	(d)		$P(A \cap B) = \frac{5}{15}$ ✓ $P(B) = \frac{7}{15}$ $\therefore P(A \mid B) = \frac{P(A \cap B)}{P(B)}$ $= \frac{\frac{5}{15}}{\frac{7}{15}}$ $= \frac{5}{7}$ ✓
	(e)		$\frac{20 \times 78 + 96 + 82.}{22} = 79$ $\therefore \mathit{New\ mean} = 79$ ✓
	(f)		Dilate vertically by a factor of 4 ✓ Shift right by 1 unit ✓ Shift up by 3 units. ✓
	(g)		$f'(x) = \frac{7}{(3x+2)^2}$ $f(x) = \frac{2x-1}{3x+2} \rightarrow u = 2x-1, u' = 2, v = 3x+2, v' = 3$ $f'(x) = \frac{vu' - uv'}{v^2}$ $= \frac{2(3x+2) - 3(2x-1)}{(3x+2)^2}$ ✓ $= \frac{6x+4-6x+3}{(3x+2)^2}$ $= \frac{7}{(3x+2)^2}$ $m(\text{tangent}) = f'(-1)$ $= \frac{7}{(3 \times -1 + 2)^2}$ $= 7$ ✓ $m(\text{normal}) = \frac{-1}{7}$ ✓ at $(-1, 3)$

Question 13

<table>
<tr><td>13.</td><td>(a)</td><td>(i)</td><td>0.6927 (4 decimal places) $\boxed{\checkmark}$</td></tr>
<tr><td></td><td></td><td>(ii)</td><td>Moderate positive correlation $\boxed{\checkmark}$</td></tr>
<tr><td></td><td></td><td>(iii)</td><td>$N = 14.300 + 0.076t$ $\boxed{\checkmark}$ y intercept
$\boxed{\checkmark}$ gradient</td></tr>
<tr><td></td><td></td><td>(iv)</td><td>$\boxed{\checkmark}$ plotting line on the axes (check it goes through t = 100 and N = 21.9)

</td></tr>
<tr><td></td><td></td><td>(v)</td><td>$N = 14.300 + 0.076t$
$27 = 14.300 + 0.076t$
$\therefore\ t = 167$ (nearest whole number) $\boxed{\checkmark}$</td></tr>
<tr><td></td><td></td><td>(vi)</td><td>27 tasks is within the domain, so therefore an example of interpolation. $\boxed{\checkmark}$</td></tr>
<tr><td></td><td>(b)</td><td>(i)</td><td><table><tr><td>X</td><td>0</td><td>1</td><td>2</td></tr><tr><td>$P(X = x)$</td><td>a</td><td>b</td><td>0.3</td></tr></table>
$E(x) = 0 \times a + 1 \times b + 2 \times 0.3 = 0.8$
$\therefore b = 0.2$ $\boxed{\checkmark}$

$a + b + 0.3 = 1$
$a + 0.2 + 0.3 = 1$
$\therefore a = 0.5$ $\boxed{\checkmark}$</td></tr>
<tr><td></td><td></td><td>(ii)</td><td>$E(X^2) = 0^2 \times 0.5 + 1^2 \times 0.2 + 2^2 \times 0.3 = 1.4$ $\boxed{\checkmark}$

$Var(X) = E(X^2) - [E(X)]^2 = 1.4 - 0.8^2 = 0.76$ $\boxed{\checkmark}$</td></tr>
</table>

13.	(c)	(i)	$D = \int_1^5 A(t)\ dt$ $\therefore D \approx \frac{5-1}{2(4)}\left[4\ln 11 + 4\ln 51 + 2\left(4\ln 21 + 4\ln 31 + 4\ln 41\right)\right]$ ☑ $D \approx 53.4277685$ ☑ $D \approx 53\,\text{cm} \quad (\textit{nearest centimetre})$
		(ii)	$\left.\begin{array}{l} A'(t) = \frac{40}{1+10t} \\ A''(t) = -\frac{400}{(1+10t)^2} \\ A''(t) < 0 \ \text{for } t \geq 0 \end{array}\right\}$ ☑ $\therefore$ under estimate as concave down ☑

Question 14

14.	(a)	(i)	$4x^2 - x^4 = 0$ $x^2\left(4 - x^2\right) = 0$ ☑ $x = 0,$ $4 - x^2 = 0$ $x = 0, \pm 2$ ☑
		(ii)	$f(x) = 4x^2 - x^4$ $f(-x) = 4(-x)^2 - (-x)^4 = 4x^2 - x^4$ ☑ Since $f(x) = f(-x)$ the function is even. ☑
		(iii)	$f(x) = 4x^2 - x^4$ $f'(x) = 8x - 4x^3$ $f'(\sqrt{2}) = 8\sqrt{2} - 4\left(\sqrt{2}\right)^3$ ☑ $= 8\sqrt{2} - 4 \times \left(2\sqrt{2}\right)$ $= 0$
		(iv)	Since the function is even it is a reflection of itself in the y-axis. So any stationary points on the right hand side of the y-axis will appear on the left hand side ☑

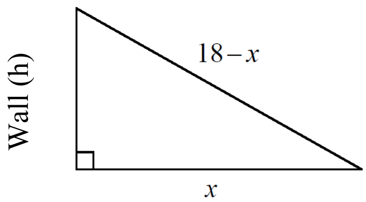
	(b)	(i)	$a=1.8\times10^{9}, r=0.9,\ n=4$ $T_4=1.8\times10^{9}\times0.9^{4-1}=1.3\times10^{9}\text{ m}^3$ ☑
		(ii)	$S_{10}=\dfrac{1.8\times10^{9}\left(0.9^{10}-1\right)}{0.9-1}$ ☑ $=1.172378808\times10^{10}\text{ m}^3$ ☑ $=1.2\times10^{10}\text{ m}^3$ (correct to 1 dp) ☑
		(iii)	$S_\infty=\dfrac{1.8\times10^{9}}{1-0.9}=1.8\times10^{10}$ ☑ $<1.9\times10^{10}$ $\therefore$ By limiting sum can never exceed 1.8×10^{10} $\therefore$ Can never exceed $1.9\times10^{10}\text{ m}^3$ ☑
	(c)		$e^{2\ln x}=4x+21$ $e^{\ln x^2}=4x+21$ ☑ $x^2-4x-21=0$ $(x-7)(x+3)=0$ since $x>0\ \therefore x=7$ ☑ *OR* $e^{2\ln x}=4x+21$ $2\ln x=\ln\left(4x+21\right)$ ☑ $\ln x^2=\ln\left(4x+21\right)$ $x^2-4x-21=0$ $(x-7)(x+3)=0$ since $x>0\ \therefore x=7$ ☑
	(d)		$\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} 3\sin x\ dx=0$ since $y=3\sin x$ is an odd function. ☑

Question 15

15.	(a)	(i)	$x = e^{t^2} - 2t^2$ $v = \frac{dx}{dt}$ ✓ $= 2te^{t^2} - 4t$ ✓
		(ii)	$2te^{t^2} - 4t = 0$ $2t\left(e^{t^2} - 2\right) = 0$ $2t = 0, e^{t^2} - 2 = 0$ $t = 0$ sec ✓ $e^{t^2} = 2$ $t^2 = \ln 2$ $t = \pm\left[\sqrt{\ln 2}\right]$ ✓ but $t \geq 0$ so $t = \sqrt{\ln 2}$ sec ✓
		(iii)	$\left.\begin{array}{l} v = 2te^{t^2} - 4t \\ acc = \frac{dv}{dt} \end{array}\right\}$ ✓ $= e^{t^2} \times 2 + 2t \times 2te^{t^2} - 4$ ✓ $= 2e^{t^2} + 4t^2e^{t^2} - 4$
	(b)	(i)	$y(t) = -3\cos\left(\frac{\pi t}{6}\right) + 15$ $\therefore a = -3$ ✓ $b = \frac{\pi}{6}$ ✓
		(ii)	$13.5 = -3\cos\left(\frac{\pi t}{6}\right) + 15$ $\frac{1}{2} = \cos\left(\frac{\pi t}{6}\right)$ ✓ $\frac{\pi t}{6} = \frac{\pi}{3}, \frac{5\pi}{3}$ $t = 2, 10$ ✓ $\therefore$ between 9 am and 5pm *OR* Reading from graph $t = 2, 10$ ✓ $\therefore$ between 9 am and 5pm ✓
	(c)	(i)	Initial population when $t = 0$ $P = 1000e^0$ $= 1000$ ✓

		(ii)	$25000 = 1000e^{10k}$ $25 = e^{10k}$ $10k = \ln 25$ $k = \dfrac{\ln 25}{10}$ $\boxed{\checkmark}$ $k = 0.3219$ (4 d.p.)
		(iii)	$80000 = 1000e^{0..3219t}$ $80 = e^{0..3219t}$ $0.3219t = \ln 80$ $\boxed{\checkmark}$ $t = \dfrac{\ln 80}{0.3219}$ $t = 13.613$ $\boxed{\checkmark}$ $\therefore$ Exceeds 80 000 during the 14th week

Question 16

16.	(a)	(i)	$T_3 = 2^3 + 3(3) = 17$ songs $\boxed{\checkmark}$
		(ii)	$S_n = 2^n \quad a = 2 \quad r = 2 \quad \therefore S_{20} = \dfrac{2\left(2^{20} - 1\right)}{2-1}$ $= 2\ 097\ 150$ songs $\boxed{\checkmark}$ $S_n = 3n \quad a = 3 \quad d = 3 \quad \therefore S_{20} = \dfrac{20}{2}\left(2\times 3 + (20-1)\times 3\right)$ $= 630\ songs$ $\boxed{\checkmark}$ $\therefore Total\ songs = 20\ 971\ 150 + 630 = 2\,097\,780$ songs $\boxed{\checkmark}$
	(b)	(i)	 $h = \sqrt{(18-x)^2 - x^2}$ $= \sqrt{324 - 36x}$ $= \sqrt{36(9-x)}$ $= 6\sqrt{9-x}$ $\boxed{\checkmark}$ $A = \dfrac{1}{2}\times x \times 6\sqrt{9-x}$ $= 3x\sqrt{9-x}$ $\boxed{\checkmark}$

		(ii)	

$$A = 3x(9-x)^{\frac{1}{2}}$$

$$\frac{dA}{dx} = 3x.\frac{1}{2}(9-x)^{-\frac{1}{2}}.-1 + (9-x)^{\frac{1}{2}}.3$$ ✓

$$= \frac{-3x}{2\sqrt{9-x}} + \frac{3(9-x)}{\sqrt{9-x}}$$

$$= \frac{-3x+6(9-x)}{2\sqrt{9-x}}$$

$$= \frac{-3x+54-6x}{2\sqrt{9-x}}$$

$$= \frac{54-9x}{2\sqrt{9-x}}$$ ✓

Maximuim occurs when $\frac{dA}{dx} = 0$

$$\frac{dA}{dx} = 0$$

$$\frac{54-9x}{2\sqrt{9-x}} = 0$$

$$x = 6 \text{ m}$$ ✓

Check maximum using table

x	5	6	7
$\frac{dA}{dx} = \frac{54-9x}{2\sqrt{9-x}}$	$\frac{9}{2\sqrt{4}}$	0	$\frac{-9}{2\sqrt{2}}$
	+	0	-

✓

Therefore maximum when $x = 6$ metres.

OR

Alternative method

$$\frac{dA}{dx}=\frac{54-9x}{2\sqrt{9-x}}$$

$u=54-9x \quad u'=-9 \quad v=2(9-x)^{\frac{1}{2}} \quad v'=(9-x)^{-\frac{1}{2}}$

$$\frac{d^2A}{dx^2}=\frac{2(9-x)^{-\frac{1}{2}}\times-9-\left((54-9x)\times-(9-x)^{-\frac{1}{2}}\right)}{\left(2(9-x)^{\frac{1}{2}}\right)^2}=\frac{-18(9-x)^{-\frac{1}{2}}+\left((54-9x)\times(9-x)^{-\frac{1}{2}}\right)}{\left(2(9-x)^{\frac{1}{2}}\right)^2}$$

$$\frac{d^2A}{dx^2}=\frac{-18\sqrt{9-x}+\frac{54-9x}{\sqrt{9-x}}}{4(9-x)}=\frac{\frac{-18\sqrt{9-x}\sqrt{9-x}}{\sqrt{9-x}}+\frac{54-9x}{\sqrt{9-x}}}{4(9-x)}$$

$$\frac{d^2A}{dx^2}=\frac{\frac{-18(9-x)+54-9x}{\sqrt{9-x}}}{4(9-x)}=\frac{\frac{-162+18x+54-9x}{\sqrt{9-x}}}{4(9-x)}$$

$$\frac{d^2A}{dx^2}=\frac{\frac{-108+9x}{\sqrt{9-x}}}{4(9-x)}=\frac{-108+9x}{\sqrt{9-x}}\div 4(9-x)$$

$$\frac{d^2A}{dx^2}=\frac{-108+9x}{\sqrt{9-x}}\times\frac{1}{4(9-x)}=\frac{-9(12-x)}{\sqrt{9-x}}\times\frac{1}{4(9-x)}$$

$$\frac{d^2A}{dx^2}=\frac{-9(12-x)}{\sqrt{9-x}\times4(9-x)}=\frac{-9(12-x)}{4(9-x)^{\frac{3}{2}}}$$

$x=6,$

$$\frac{d^2A}{dx^2}=\frac{-9(12-6)}{4(9-6)^{\frac{3}{2}}}=-2.5980....$$

$\therefore \frac{d^2A}{dx^2}<0, \therefore$ max value at $x=6$ √

√

√

(c)

$$\frac{dV}{dt}=R=-k$$

$V=-kt+c$

when $t=0,\ V=64$ ✓

$64=0+c$

$\therefore V=64-kt$ ✓

when $t=9,\ V=1$

$1=64-9k$

$9k=63$

$k=7$ ✓

$\therefore V=64-7t$

OR

			When $t=0,\ V=64$ *and* $t=9,\ V=1$ *Difference is* 63m^3 $\boxed{\checkmark}$ Since rate of change is constant over change in time (given) $\boxed{\checkmark}$ $\dfrac{64-1}{9-0}=7$ $\therefore k=7$ $\boxed{\checkmark}$ *then* $V=64-7t$
		(ii)	If cube has edge length 2 m, volume will be $8\,\text{m}^3$ $\boxed{\checkmark}$ $8=64-7t$ $7t=56$ $t=8$ The cube will have length of 2 m after 8 hours. $\boxed{\checkmark}$